Finding the Keys to Peano Curve

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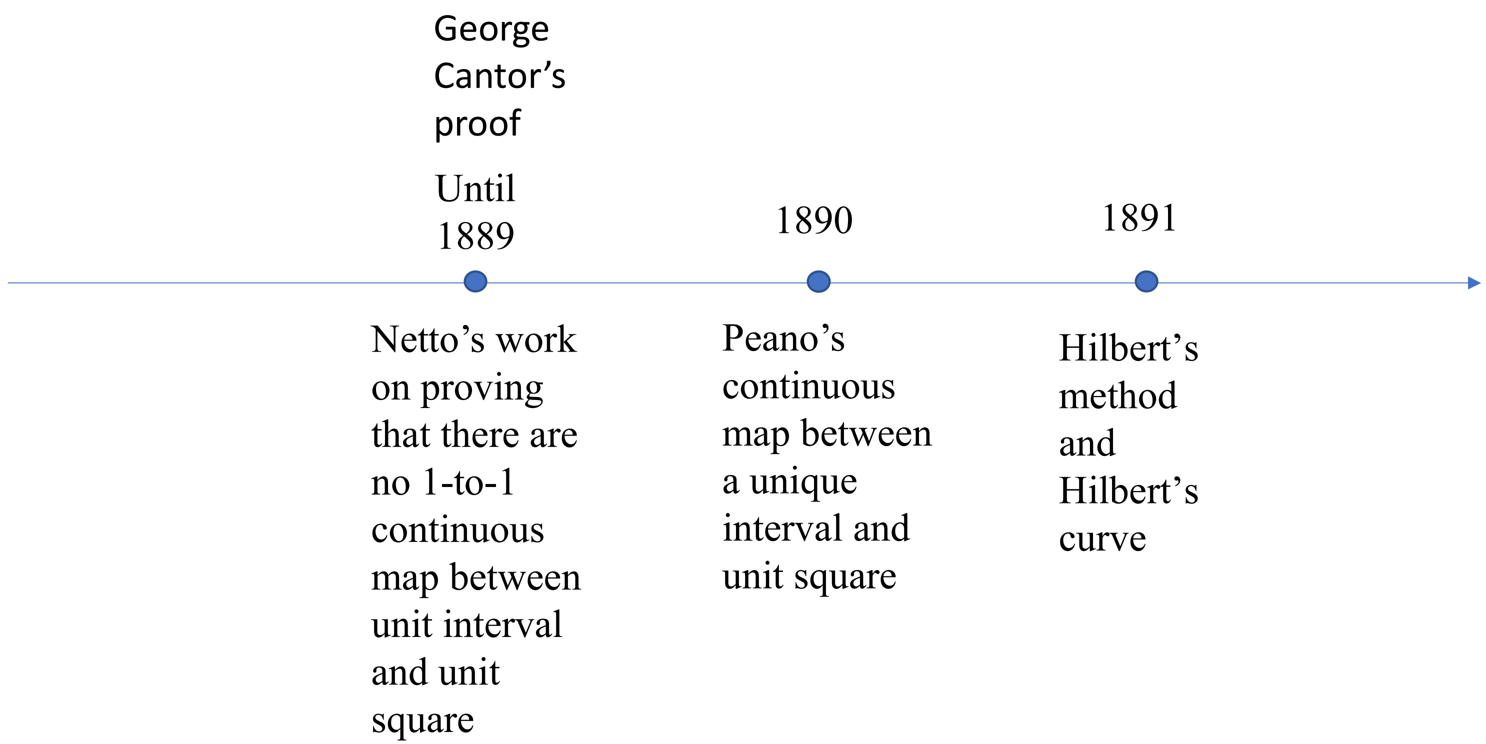
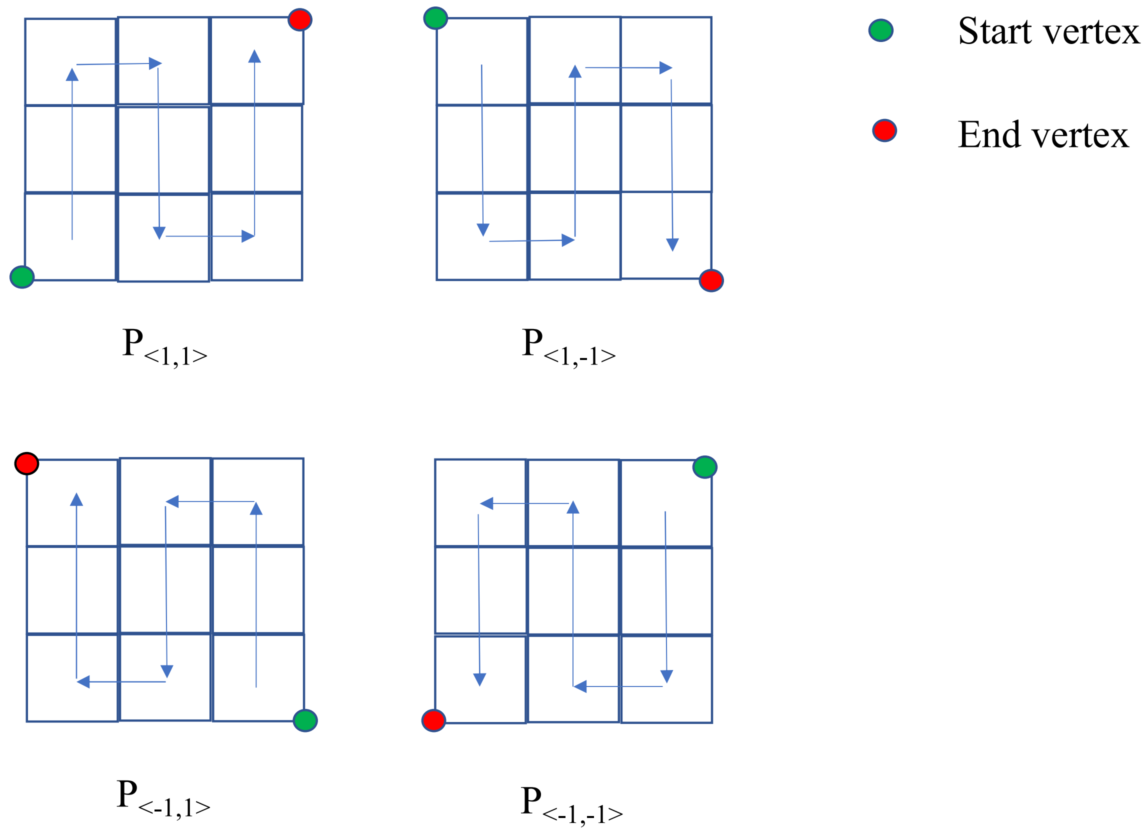
I. Historical Context

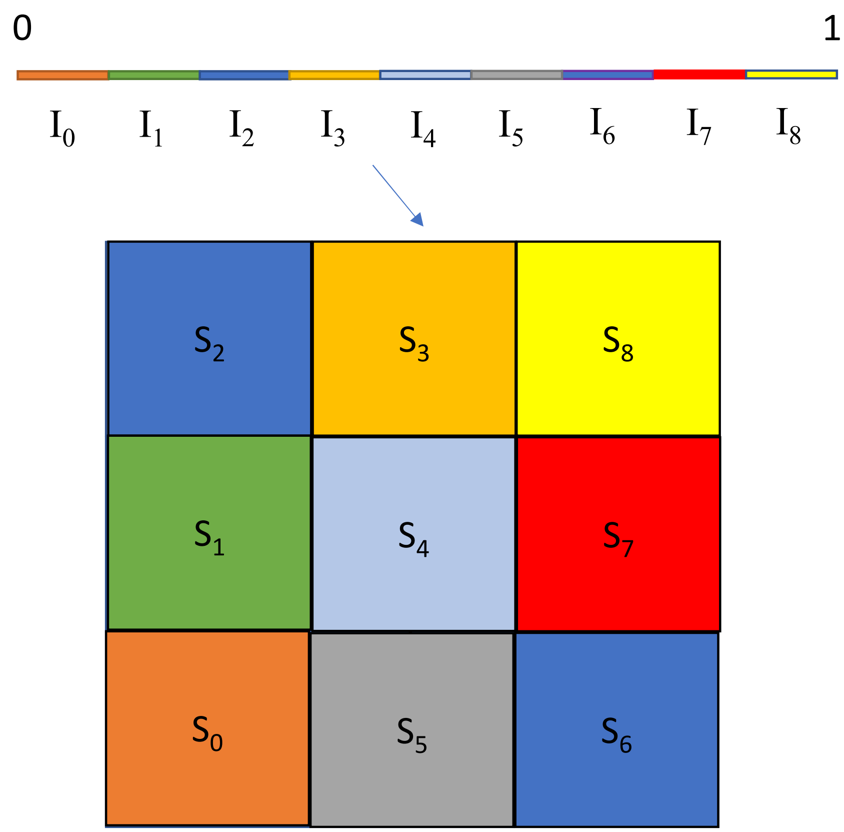


II. Construction idea



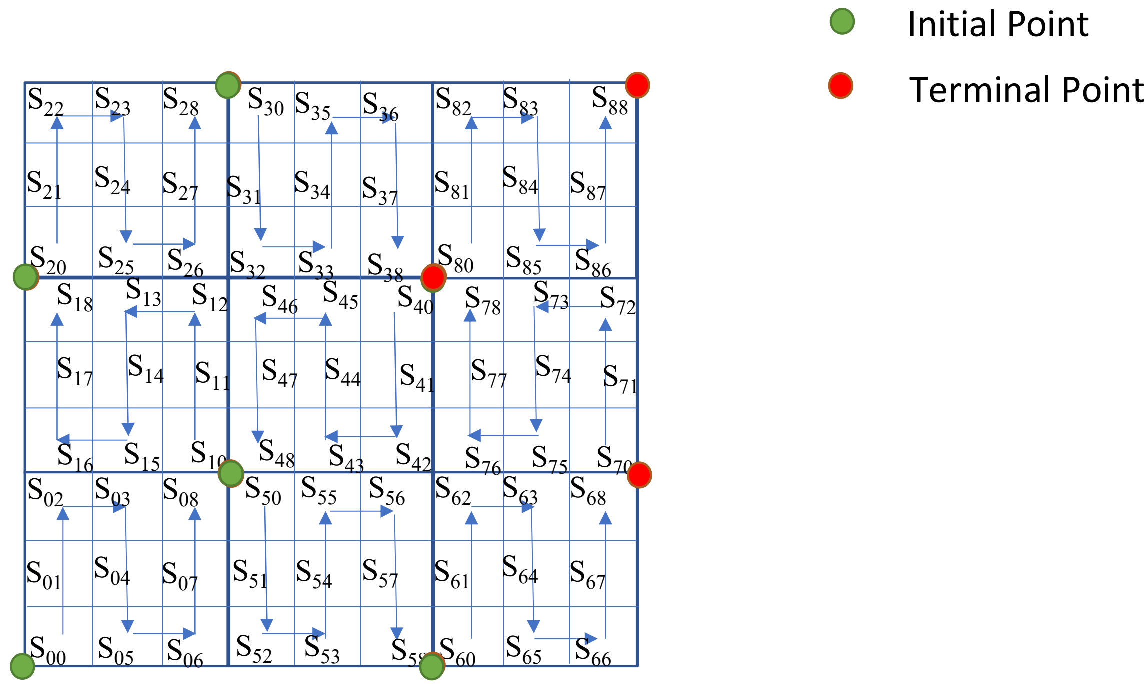
The four Peano Patterns

III. Inductive construction



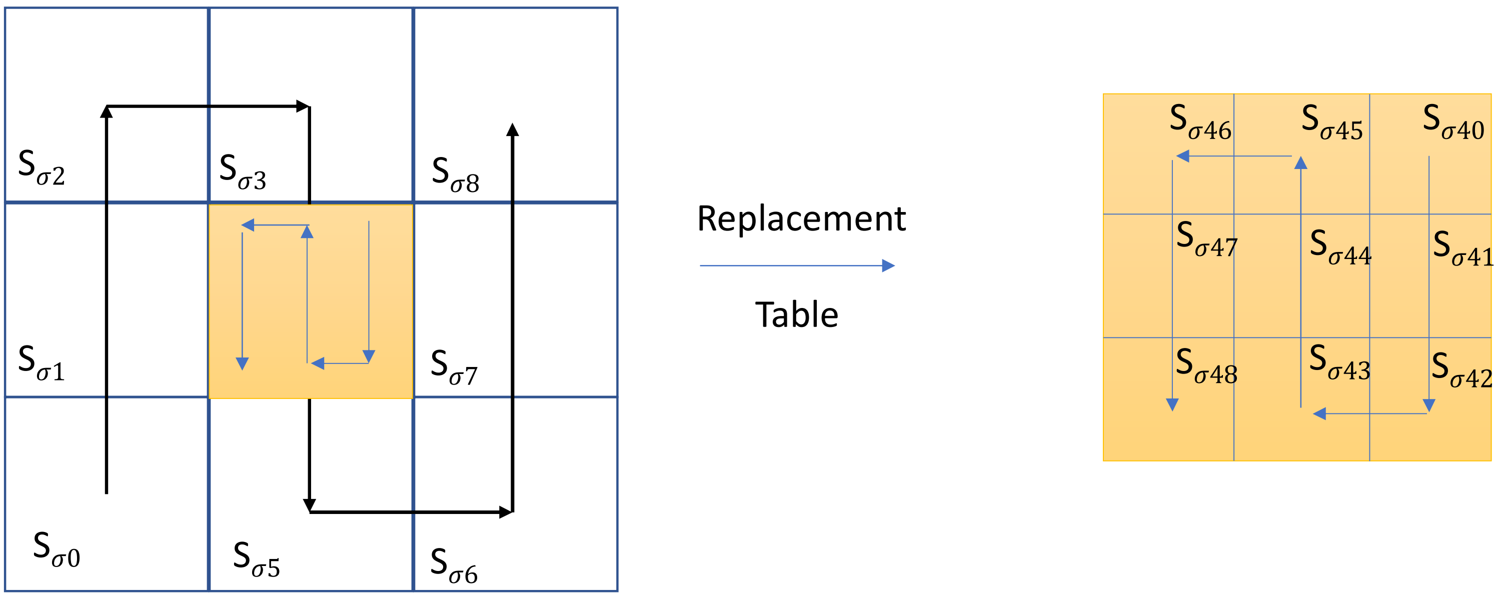
First stage of Peano Curve
(The initial Pattern and its first numbering)

III. Inductive construction



Second stage of Peano Curve

III. Inductive construction



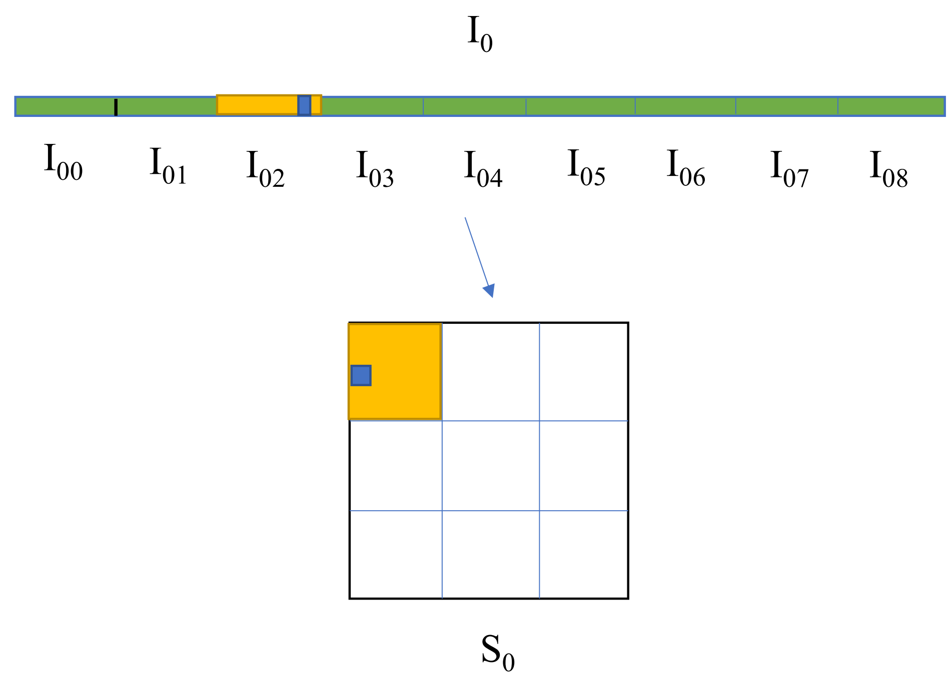
Level n triadic subsquare S_{σ}

Level n+1 triadic subsquare $S_{\sigma 4}$

IV. Specific Example

Suppose $x = 0.027\ldots_9$

$x = I_0 \cap I_{02} \cap I_{027} \cap \ldots$ and $f_P(x) = S_0 \cap S_{02} \cap S_{027} \cap \ldots$



V. Our Results

1. We can use this base 9 identification of points in [0,1] to points of S to determine if a point is 1-to-1, 2-to-1 or 4-to-1 point

2. We prove that there are no 3-to-1 points

3. We can prove that the replacement table is generated by the action of the Klein 4-group.

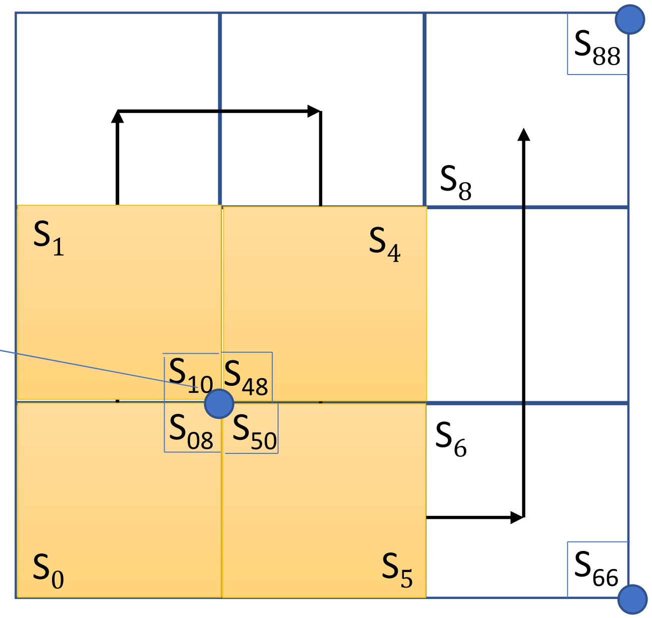
$(1,1) = S_8 \cap S_{88} \cap S_{888} \cap \ldots \rightarrow (1,1) = f_P(0.888.._9)$

$(1/3,1/3) = f_P(0.088.._9) = S_0 \cap S_{08} \cap S_{088} \cap \ldots$

$(1/3,1/3) = f_P(0.100.._9) = S_1 \cap S_{10} \cap S_{100} \cap \ldots$

$(1/3,1/3) = f_P(0.488.._9) = S_4 \cap S_{48} \cap S_{488} \cap \ldots$

$(1/3,1/3) = f_P(0.500.._9) = S_5 \cap S_{50} \cap S_{500} \cap \ldots$



The level 1 square

$(1,0) = S_6 \cap S_{66} \cap S_{666} \cap \ldots$

$(1,0) = f_P(0.666.._9)$

VI. References

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